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General Certificate of Education (A-level)
June 2013

Mathematics MFP1

(Specification 6360)

Further Pure 1

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$(x_2 =)\ 10 - \dfrac{\left(10^3 - 10^2 + 4\times 10 - 900\right)}{\left(3\times 10^2 - 2\times 10 + 4\right)}$	B1		$10 - \dfrac{\mathrm{f}(10)}{\mathrm{f}\,'(10)}$ with a correct numerical expression or value PI for f(10).
	$\left(\begin{array}{l} = 10 - \dfrac{1000-100+40-900}{300-20+4} \\ = 10 - \dfrac{40}{284} = 10 - 0.1408... \end{array}\right)$	B1		$10 - \dfrac{\mathrm{f}(10)}{\mathrm{f}\,'(10)}$ with a correct numerical expression or value PI for f′(10).
	(= 9.85915….) = 9.859 (to 4 sf)	B1	3	Must be 9.859
	Total		**3**	
2(a)(i)	$\mathbf{A} - \mathbf{B} = \begin{bmatrix} p-3 & 1 \\ 2 & p-3 \end{bmatrix}$	B1	1	
(ii)	$\mathbf{AB} = \begin{bmatrix} p & 2 \\ 4 & p \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3p+4 & p+6 \\ 12+2p & 4+3p \end{bmatrix}$	M1		Finding **AB** and at least 2 elements correct
		A1	2	CSO
(b)	$\mathbf{A}-\mathbf{B}+\mathbf{AB} = \begin{bmatrix} 4p+1 & p+7 \\ 14+2p & 1+4p \end{bmatrix}$	B1F		Only ft if all matrices are 2 by 2 PI by later correct work
	$\mathbf{A}-\mathbf{B}+\mathbf{AB} = k\,\mathbf{I} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1		**I** used as or equated to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ at some stage
	$(p+7=0\ ,\ 14+2p=0\ \ \Rightarrow)\ p=-7$	B1		$p=-7$ provided it gives the relevant two zero elements
	$p=-7 \Rightarrow \mathbf{A}-\mathbf{B}+\mathbf{AB} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} = -27\,\mathbf{I}$ $\Rightarrow\ k=-27$	B1	4	CSO Either −27**I** (no earlier errors) for B1 OR $k=-27$ with either $\begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$ or $27\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ or $-27\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ seen before (no earlier errors) for B1
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)	$\cos\left(5x+40^\circ\right)=\cos 65^\circ$ $5x + 40^\circ = \pm 65^\circ$	B1		Both $\pm 65^\circ$ OE eg $5x + 40 = 65, 295$
	$5x+40^\circ = 360n^\circ + 65^\circ$, $5x+40^\circ = 360n^\circ - 65^\circ$	M1		$5x+40 = 360n \pm \alpha$ Either one, OE Condone $2n\pi$ for $360n$
	$x = \dfrac{360n^\circ + 65^\circ - 40^\circ}{5}$, $x = \dfrac{360n^\circ - 65^\circ - 40^\circ}{5}$	m1		Either one, OE Correct rearrangement of $5x+40 = 360n \pm \alpha$ OE to $x =$. Condone $2n\pi$ for $360n$
	$x = 72n^\circ + 5^\circ$, $x = 72n^\circ - 21^\circ$	A2,1,0	5	OE Full set of correct solns. in degrees written in a simplified form. (A1 if not in a simplified form) (A0 if radians present in answer)
(b)	$\dfrac{\sqrt{3}-1}{2\sqrt{2}} = (\cos\dfrac{\pi}{4})\left[\cos(a\pi)+\cos(b\pi)\right]$ $\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\right) = \dfrac{1}{\sqrt{2}}\left[\cos(a\pi)+\cos(b\pi)\right]$	B1		Recognising $\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$ (or $= \dfrac{1}{\sqrt{2}}$) PI eg by seeing $\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\right)$
	$\cos a\pi = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6}$, $(a = \dfrac{1}{6})$	B1		OE ie any correct rational value for a which satisfies $\cos a\pi = \dfrac{\sqrt{3}}{2}$
	$\cos b\pi = -\dfrac{1}{2} = \cos\dfrac{2\pi}{3}$, $(b = \dfrac{2}{3})$	B1	3	OE ie any correct rational value for b which satisfies $\cos b\pi = -\dfrac{1}{2}$
	$\sin\dfrac{\pi}{12} = \cos\left(\dfrac{\pi}{4}\right)\left[\cos\left(\dfrac{1}{6}\pi\right)+\cos\left(\dfrac{2}{3}\pi\right)\right]$			Note: labels a and b could be interchanged.
	Total		**8**	

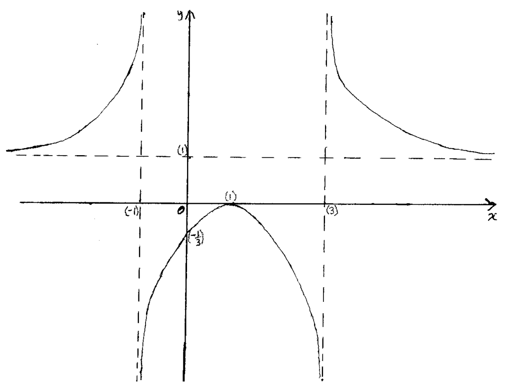
Q	Solution	Marks	Total	Comments
4(a)(i)	$(z-2\mathrm{i})^* = (x+y\,\mathrm{i}-2\mathrm{i})^* = x+(2-y)\mathrm{i}$	B1	1	$x + 2\mathrm{i} - y\mathrm{i}$ OE rearrangement
(ii)	$(z-2\mathrm{i})^* = 4\mathrm{i}\,z+3 = 4\mathrm{i}x+4\mathrm{i}^2y+3 = 4\mathrm{i}x-4y+3$	B1		$\mathrm{i}^2=-1$ used
	$x+(2-y)\,\mathrm{i} = 4\mathrm{i}x-4y+3$ (#) Real parts: $x=-4y+3$			Attempting to equate, without mixing real and imaginary terms, **both** the real parts and the imag. parts for the c's eqn (#).
	Imaginary parts: $2-y=4x$	M1		
		A1F		If not corrected, ft on [c's(a)(i)] $= 4\mathrm{i}x - 4y + 3$ provided both the resulting linear equations have non zero x, y and const terms
	$y=\dfrac{2}{3}$, $x=\dfrac{1}{3}$	A1		Solving correct equations, to obtain either $x=\dfrac{1}{3}$ OE or $y=\dfrac{2}{3}$ OE
	$(z=)\ \dfrac{1}{3}+\dfrac{2}{3}\mathrm{i}$	A1	5	$\dfrac{1}{3}+\dfrac{2}{3}\mathrm{i}$
(b)	(One of the) coefficients (of the quadratic equation is) not real.	E1	1	OE eg Sum of roots is $-10\,\mathrm{i}$ so p cannot be real if roots are $p\pm q\mathrm{i}$
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$y = 2x^2 - 5x$			
	$y_Q = 2(1+h)^2 - 5(1+h) = 2 + 4h + 2h^2 - 5 - 5h$ $(= 2h^2 - h - 3)$	B1		$y_Q = 2(1+h)^2 - 5(1+h)$ with correct expansion of brackets PI.
	Grad. $= \dfrac{y_Q - y_P}{x_Q - x_P} = \dfrac{2(1+h)^2 - 5(1+h) - (-3)}{1 + h - 1}$	M1		Use of correct formula for gradient
	$= \dfrac{2h^2 - h - 3 - (-3)}{h} = \dfrac{2h^2 - h}{h} = 2h - 1$	A1	3	CSO
(ii)	As $h \to 0$, (grad of $PQ \to$ grad of tangent at P) (ie) gradient (of tangent at P) $= -1$	E1		$h = 0$ scores E0
	Now **gradient** of $x+y=0$ (or $y = -x$) is also **−1** $\Rightarrow$ tangent at P is **parallel** to line $x + y = 0$	E1	2	Dep on $h \to 0$ or $h = 0$ being used earlier
(b)	$I = \int_1^{\infty} x^{-4}\left(2x^2 - 5x\right)\,dx = \int_1^{\infty}\left(2x^{-2} - 5x^{-3}\right)\,dx$			
	$I = \left[-2x^{-1} - 5\dfrac{x^{-2}}{-2}\right]_1^{\infty}$	M1		At least one term correct
	As $x \to \infty$, $x^{-1} \to 0$ and $x^{-2} \to 0$	E1		OE Ft on $k\,x^{-n}$ provided M1 awarded
	$I = 0 - (-2 + 5/2) = -\dfrac{1}{2}$	A1	3	(I =) $-\dfrac{1}{2}$ Dep on both terms integrated correctly in the M1 line
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)	$\alpha+\beta=-\frac{3}{2}$	B1		OE
	$\alpha\beta=-3$	B1	2	OE
(b)	$\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		Using correct identity for $\alpha^3+\beta^3$ in terms of $\alpha+\beta$ and $\alpha\beta$.
	$=\left(-\frac{3}{2}\right)^3-3(-3)(-3/2)$	A1F		with ft/or correct substitution
	$=-\frac{27}{8}-\frac{27}{2}=-\frac{135}{8}$	A1	3	CSO AG. Correct evaluation of each of $(-1.5)^3$ and $-3(-3)(-1.5)$ must be seen before the printed answer is stated
(c)	Sum = $\alpha+\frac{\alpha}{\beta^2}+\beta+\frac{\beta}{\alpha^2}$ $=\alpha+\beta+\frac{\alpha^3+\beta^3}{(\alpha\beta)^2} = -\frac{3}{2}+\frac{-135/8}{9}$	M1		Writing $\alpha+\frac{\alpha}{\beta^2}+\beta+\frac{\beta}{\alpha^2}$ in a suitable form with ft/or correct substitution
	Sum $=-\frac{27}{8}$	A1		PI OE exact value eg -3.375 (A0 if $\alpha\beta=3$ used to get $(\alpha\beta)^2=9$)
	Product = $\alpha\beta+\frac{\beta}{\alpha}+\frac{\alpha}{\beta}+\frac{1}{\alpha\beta}$			
	$=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$ **(*)** Now $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ **(**$=\frac{9}{4}+6$**)**	M1		**(*)** OE with correct identity for $\alpha^2+\beta^2$ used in (c). Subst of values not required but PI by correct value of Product
	Product = $-3-\frac{1}{3}\left(\frac{9}{4}+6\right)-\frac{1}{3}=-\frac{73}{12}$	A1		PI OE exact value
	x^2-Sx+P $(=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.
	Eqn is $24x^2+81x-146=0$	A1	6	OE but integer coefficients and '= 0' needed
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$f(x) = 4x^3 - x - 540\,000$ $f(51) = -9447$ (<0); $f(52) = 22380$ (>0); Since sign change (and f continuous), $51<\alpha<52$	M1 A1	2	f(51) and f(52) both considered All values and working correct plus relevant concluding statement involving '51' and '52'.
(b)(i)	$S_n = \sum_{r=1}^{n}(2r-1)^2 = \sum 4r^2 - \sum 4r + \sum 1$	M1		Splitting up the sum into separate sums. PI by m1 line below or better
	$= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + \sum_{r=1}^{n} 1$	m1		Substitution of correct formulae from FB for the two summations
	$= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + n$	B1 A1		B1 for $\sum_{r=1}^{n} 1 = n$ stated or used
	$= \frac{n}{3}\left[2(2n^2+3n+1) - 6(n+1) + 3\right] = \frac{n}{3}\left[4n^2-1\right]$	A1	5	CSO
(ii)	$(6S_n = 2n[4n^2-1]) = 2n(2n-1)(2n+1)$ $(2n-1)$, $2n$ and $(2n+1)$ are consecutive integers	B1 E1	2	Terms in any order Terms must be identified and statement 'consecutive integers'
(c)	$S_n = 1^2 + 3^2 + 5^2 + \ldots. + (2n-1)^2$ ie sum of squares of first n odd numbers so need least N such that $S_N > 180\,000$			
	$S_{52} = \frac{52}{3}\left[4\times 52^2 - 1\right] = 187460$ and $S_{51} = 176851$	M1		Either $\frac{n}{3}\left[4n^2-1\right] = 180000$ or $2N(2N-1)(2N+1) = 1080000$ or S_{52} and S_{51} both attempted (or = replaced by > or by ≥)
	Smallest value of N is 52	A1	2	CSO Fully and correctly justified. NMS N=52 scores 0/2
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$	M1		Matrix in form $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$, where $\lambda \neq 0$, $\mu \neq 0$ and $\lambda \neq \mu$
		A1	2	$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$
(b)(i)	$y = \sqrt{3}\, x = \tan 60^{\circ}\, x$ $\quad \begin{bmatrix} \cos 120^{\circ} & \sin 120^{\circ} \\ \sin 120^{\circ} & -\cos 120^{\circ} \end{bmatrix}$	M1		$\begin{bmatrix} \cos 120 & \sin 120 \\ \sin 120 & -\cos 120 \end{bmatrix}$ PI For M mark, condone dec approx 0.86 or 0.87 or better in place of sin120º
	Required matrix is $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	A1	2	OE but must be in exact surd form.
(ii)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \ldots\ldots$	M1		Attempt to multiply c's (b)(i) 2by2 matrix and c's (a) 2by2 matrix in **correct** order.
	$= \begin{bmatrix} -\frac{1}{2} & \frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$	A1	2	OE but must be in exact surd form.
	Total		**6**	

Q	Solution	Marks	Total	Comments
9(a)	(HA) $y = 1$	B1		$y = 1$ OE eqn
	(VA) $x^2 - 2x - 3 = 0 \quad (x-3)(x+1) = 0$	M1		PI OE eg use of quadratic formula
	$x = -1$ and $x = 3$	A1	3	Both needed OE eqn(s)
(b)(i)	$k = \dfrac{x^2 - 2x + 1}{x^2 - 2x - 3} \Rightarrow kx^2 - 2kx - 3k = x^2 - 2x + 1$ $kx^2 - 2kx - 3k - x^2 + 2x - 1 = 0$ $(k-1)x^2 - 2(k-1)x - (3k+1) = 0$	B1	1	AG Must see the two stages, correct elimination of fraction and a correct rearrangement to … = 0, along with correct elimination of brackets before printed answer is stated.
(ii)	Discriminant $b^2 - 4ac$ $\{\, 4(k-1)^2 + 4(k-1)(1+3k) \,\}$	M1		$b^2 - 4ac$, OE, in terms of k; condoning one minor error in substitution.
	Line intersects curve $\Rightarrow b^2 - 4ac \geq 0$ $\Rightarrow 4(k-1)^2 + 4(k-1)(1+3k) \geq 0$	A1		A correct inequality where k is the only unknown
	$\Rightarrow 4(k-1)[k - 1 + 1 + 3k] \geq 0, \quad 16k(k-1) \geq 0$ ie $k^2 - k \geq 0$	A1	3	CSO AG Must be convinced
(iii)	$k^2 - k \geq 0, \quad k(k-1) \geq 0,$ $k \leq 0, \quad k \geq 1$ Critical values $k = 0$, $(k = 1)$	B1		For $k = 0$ either as an equation or inequality.
	$k \neq 1$ since there is no point on the curve where y=1 ($x^2 - 2x - 3 \neq x^2 - 2x + 1$)	E1		OE Valid explanation, with no accuracy errors, to discount k=1
	k=0, $-x^2 + 2x - 1 = 0$ or $y = 0$, $x^2 - 2x + 1 = 0$	M1		OE
	(Only one) **stationary** point (and its coordinates are) **(1, 0)**	A1	4	'stationary' with either (1, 0) or $\{x$=1, $y = 0\}$
(c)		B1		Curve with three distinct branches
		B1		Branch between VAs, correct shape, no part of the branch above the x-axis, only intersection with y-axis at a point below the origin, and its max pt on the positive x-axis
		B1	3	Fully correct curve drawn with each branch correctly approaching its relevant asymptotes
	Total		**14**	
	TOTAL		**75**	